# ALGEBRA II (answer)

by Andrew Becker

In order to solve the given equations, it's necessary to know how to multiply and add two numbers. So the first step is to construct the addition and multiplication tables for the numbers.

From the rules we know x + **O** = x, x * **I** = x, x + x = **O**, and from the handout we know that x * **O** = **O.** This gives:

| + | O | I | A | W | M | H | T | R |
|---|---|---|---|---|---|---|---|---|
| O | O | I | A | W | M | H | T | R |
| I | I | O | ? | ? | ? | ? | ? | ? |
| A | A | ? | O | ? | ? | ? | ? | ? |
| W | W | ? | ? | O | ? | ? | ? | ? |
| M | M | ? | ? | ? | O | ? | ? | ? |
| H | H | ? | ? | ? | ? | O | ? | ? |
| T | T | ? | ? | ? | ? | ? | O | ? |
| R | R | ? | ? | ? | ? | ? | ? | O |

| * | O | I | A | W | M | H | T | R |
|---|---|---|---|---|---|---|---|---|
| O | O | O | O | O | O | O | O | O |
| I | O | I | A | W | M | H | T | R |
| A | O | A | ? | ? | ? | ? | ? | ? |
| W | O | W | ? | ? | ? | ? | ? | ? |
| M | O | M | ? | ? | ? | ? | ? | ? |
| H | O | H | ? | ? | ? | ? | ? | ? |
| T | O | T | ? | ? | ? | ? | ? | ? |
| R | O | R | ? | ? | ? | ? | ? | ? |

There are various handwritten addition and multiplication facts on the sheet. Remembering that x+y=y+x, x*y=y*x gives:

| + | O | I | A | W | M | H | T | R |
|---|---|---|---|---|---|---|---|---|
| O | O | I | A | W | M | H | T | R |
| I | I | O | ? | ? | ? | ? | ? | ? |
| A | A | ? | O | I | ? | ? | ? | ? |
| W | W | ? | I | O | ? | ? | ? | H |
| M | M | ? | ? | ? | O | ? | ? | A |
| H | H | ? | ? | ? | ? | O | ? | ? |
| T | T | ? | ? | ? | ? | ? | O | ? |
| R | R | ? | ? | H | A | ? | ? | O |

| * | O | I | A | W | M | H | T | R |
|---|---|---|---|---|---|---|---|---|
| O | O | O | O | O | O | O | O | O |
| I | O | I | A | W | M | H | T | R |
| A | O | A | ? | ? | ? | I | ? | ? |
| W | O | W | ? | ? | ? | ? | A | ? |
| M | O | M | ? | ? | ? | ? | I | ? |
| H | O | H | I | ? | ? | ? | R | ? |
| T | O | T | ? | A | I | R | ? | M |
| R | O | R | ? | ? | ? | ? | M | ? |

Applying the rules given to facts already known will allow you to generate new facts. For example:

**x + y = z implies x + z = y**

**values in the same column or row in the addition table must be different (same with nonzero columns or rows in the multiplication table**

**Having all of column T in the multiplication table gives you the whole table**

The full addition and multiplication tables are:

| + | O | I | A | W | M | H | T | R |
|---|---|---|---|---|---|---|---|---|
| O | O | I | A | W | M | H | T | R |
| I | I | O | W | A | H | M | R | T |
| A | A | W | O | I | R | T | H | M |
| W | W | A | I | O | T | R | M | H |
| M | M | H | R | T | O | I | W | A |
| H | H | M | T | R | I | O | A | W |
| T | T | R | H | M | W | A | O | I |
| R | R | T | M | H | A | W | I | O |

| * | O | I | A | W | M | H | T | R |
|---|---|---|---|---|---|---|---|---|
| O | O | O | O | O | O | O | O | O |
| I | O | I | A | W | M | H | T | R |
| A | O | A | M | R | W | I | H | T |
| W | O | W | R | H | T | M | A | I |
| M | O | M | W | T | R | A | I | H |
| H | O | H | I | M | A | T | R | W |
| T | O | T | H | A | I | R | W | M |
| R | O | R | T | I | H | W | M | A |

Using the above tables to solve for the problems given yields:

**1)** **A**<br>
2) y = **R**<br>
**3)** **O**<br>
4) x = **O**<br>
y = **M**<br>
z = **W**<br>
**5)** **I**<br>
6) x = **T**<br>
**7)** **H**<br>
8) x = **A**

i.e. **A ROOM WITH A.**

Since this is an Invader Zim themed puzzle, the answer is of course **MOOSE**.

Solvers who submitted **VIEW** instead were given a hint that (hopefully) led them to search for the Invader Zim-appropriate answer.

**$x + y = z$ implies $x + z = y$:**

| | |
|---|---|
| $x+y = z$ | Adding z to both sides: |
| $x + y + z = z+z$ | $z+z = \mathbf{O}$ (Autonegativity) |
| $x + y + z = \mathbf{O}$ | Adding y to both sides: |
| $x + y + z + y = \mathbf{O} + y$ | Additive Identity |
| $x + y + z + y = y$ | Associativity of addition |
| $x + (y + z) + y = y$ | Commutativity of addition |
| $x + (z + y) + y = y$ | Associativity of addition |
| $x + z + (y + y) = y$ | $y + y = \mathbf{O}$ (Autonegativity) |
| $x + z + \mathbf{O} = y$ | Additive Identity |
| $x + z = y$ | |

**$x + y = x + z$ implies $y = z$**

**(i.e. values in the same column or row in the addition table must be different)**

| | |
|---|---|
| $x + y = x + z$ | Add x to both sides |
| $x + x + y = x + x + z$ | $x+x = \mathbf{O}$ (Autonegativity) |
| $\mathbf{O} + y = \mathbf{O} + z$ | Additive Identity |
| $y = z$ | |

**$x*y = x*z$ implies $x = \mathbf{O}$ or $y = z$**

**(i.e. values in the same nonzero column or row in the multiplication table must be different)**

| | |
|---|---|
| $x * y = x * z$ | Multiply both sides by $x^{-1}$ |
| $x^{-1} * x * y = x^{-1} * x * z$ | Multiplicative inverse |
| $y = z$ | |

**Having all of column T in the multiplication table gives you the whole table:**

Once you know what **T***everything is, you can figure out what **W** times anything is:

For example:

| | |
|---|---|
| **W*****A** = | |
| (**T*****T**)***A**= | (since **W** = **T*****T**) |
| **T***(**T*****A**) = | (Associativity of multiplication) |
| **T*****H** = | (since **T*****A** = **H**) |
| **R** | (since **T*****H** = **R**) |

Similarly **A**=**W*****T**,so **A***x = **W***(**T***x), **H** = **A*****T**, **R** = **H*****T**, so you can figure out the full table.